REPORT NO. 632/6

HEAT TREATMENT

Quenching of Hollow Cylinders

John H. Hollomon
1st Lt., Ordnance Dept.

Clarence Zener
Physicist

February 5, 1943

WATERTOWN ARSENAL
WATERTOWN, MASS.

HEAT TREATMENT

Quenching of Hollow Cylinders

OBJECT

To develop a method of determining the conditions under which hollow steel cylinders will harden throughout.

SUMMARY OF RESULTS

1. The calculations of Grossmann of sizes of round bars and of flat plates having the same minimum hardness when quenched in different media have been modified by the introduction of a new basis for comparison.

2. The method of Grossmann for calculating the alloying elements necessary to harden a round bar throughout is extended to hollow cylinders by calculating for all quenching media the diameter of the round having the same cooling rate at the center as the minimum cooling rate in the wall of a hollow cylinder.

3. The amount of alloying elements necessary to harden the equivalent round throughout may be determined empirically by quenching round bars of different compositions and of varying sizes in the pre-elected medium or they may estimated by the

method of Grossmann.

4.   Experimentally determined sizes of solid cylinders having the same minimum hardness as hollow cylinders quenched only on the outside agree with the theory within 6 percent.

5.   For the particular conditions of the experiments in which water was forced up the inside of small cylinders, the experimentally determined sizes of solid cylinders having the same minimum hardness as the hollow cylinders were 15 percent smaller than predicted by the theory in which the severity of quench on the inside and outside were assumed to be the same.

6.   It may be inferred that the inside of the hollow cylinders of these experiments were quenched more effectively than the outside.

J. H. Hollomon
1st Lt., Ordnance Dept.

C. Zener
Physicist

APPROVED:

H. H. ZORNIG
Colonel, Ordnance Dept.
Director of Laboratory

INTRODUCTION

The method of Grossmann for calculating the effect
of alloying elements on the hardenability of rounds
has been described in detail in the literature.[1 to 8]
After calculating the multiplicative effects of the
alloying elements, a series of curves can be determined
for the effect of carbon content and grain size[2] on
the diameter which would just harden throughout in an
ideal quenching medium. In an ideal quenching medium
the surface of the round is maintained at the temper-
ature of the medium. This is approximated in practice
by agitated or sprayed brine. In an actual quench,
the diameter of the round which will just harden through-
out is the smallest one having no "unhardened core"
visible in either the fracture or the etch test. The
center of such a hardened round is approximately
50 percent martensitic. After estimating the ideal
diameter for some composition, the actual size of round
which would just harden throughout may be determined
from a series of curves for different quenching media
which relate the actual size of round to this ideal
diameter.

A series of curves prepared in a similar manner

1.-References to be found at end of report.

is presented herein in which the ideal round diameter
equivalent to a hollow cylinder is plotted as a function
of the severity of quench and wall ratio. From this
equivalent ideal round diameter the alloying elements
necessary to harden the hollow cylinder throughout may
be calculated. The method outlined by Grossmann is
admittedly only an approximate method of determining
the composition having the desired hardenability, but
it is the best method available for prediction. Knowing
the size of round which will harden throughout (either
by experiment or by estimation from Grossmann's data),
the size of the equivalent hollow cylinder may be
calculated readily. On the other hand if the size of
the hollow cylinder is known the round size which must
be hardened throughout in the given medium may also be
calculated.

DISCUSSION AND RESULTS

A. Theoretical

In order to calculate the diameter of the solid
steel cylinder which when quenched in a given medium
will have the same hardness at its center as the
minimum hardness in the wall of a hollow cylinder
quenched in the same medium, it is first necessary
to determine the conditions of time and temperature
during cooling for which the hardness at the two
positions of the two steel shapes will be the same.

-2-

Grossmann, in comparing the hardness distribution from
the center to outside of solid cylinders found that
the time required to reach half the difference between
the quenching and final temperature during cooling
determined the hardness; that is, if the outside of
one solid cylinder of a given steel composition re-
quired the same time to cool half way from the quenching
temperature to room temperature as the inside of some
other cylinder of the same composition, the hardnesses
at the specified positions would be very nearly the
same. It was noted that the shapes of the cooling
curves at different positions in solid rounds are quite
dissimilar. If the cooling rate at the center of one
bar is the same as that of the outside of another,
the "half temperature times" will be widely different.
Since the time spent below the critical temperature
before transformation is of importance in determining
the precise products of transformation, in comparisons
where the initial parts of the cooling curves are of
different shapes the time to reach half temperature is
probably a better criterion than that of cooling rate.
Grossmann and his collaborators[2] presented some ex-
perimental evidence that cooling time is the more appli-
cable criterion of hardenability when positions other
than the centers of round sections are compared and
assumed that this condition is also the more applicable

-3-

in comparing the cooling at the centers of different
round bars.

In certain important cases, the rate of cooling
in the appropriate temperature range may be specified
by a single parameter, c, in the manner indicated
below:

$$\frac{dT}{dt} = -cT, \qquad (1)$$

where T is the temperature of the element referred to
that of the quenching medium. Throughout the temper-
ature range in which Eq. (1) is obeyed a plot of T
vs. t on semi-log paper will yield a straight line.
Such a plot is presented as Fig. 1 for the centers of
solid cylinders from the data of Russell.[10] As the
quenching proceeds the curves approach straight lines.
The less severe the quench or rather the smaller the
product of H (severity of quench) times D (diameter of
the round) the more rapidly is the asymptote approached.
For each different product, HD, in this graph the
diameter (D) of the bar has been chosen which renders
the various asymptotes parallel. The centers of the
bars represented in Fig. 1 quenched in different media
have the same "c" while their half temperature times
are quite different, having the ratio 2.4:3.0:4.0 for
HD equal to 0,1, and ∞ respectively. Since most of the
differences in half temperature times occurs before the

-4-

critical temperature is reached, it is not significant. For this example the quenching temperature was assumed to be 1700° F, the critical temperature, 1600° F, and the temperature of the nose of the "S" curve 1050° F. The half temperature time starting from the critical rather than the initial temperature are in the ratio of 2.1:2.1:2.4 which is not far from the ratio 1:1:1. Since the difference between the initial and critical temperatures is variable and very difficult to fix in actual quenching practice it may be concluded that when center hardnesses are to be compared the slopes of the linear portion of the curves of Fig. 1 are a better measure of the hardenability than the half temperature times. Using this new criterion the ratio of the round diameter quenched in a medium of quench severity H D_{SH} to the round diameter quenched in an ideal medium ($D_{S\infty}$) is plotted as a function of H D_{SH} as curve a in Fig. 2. This curve is to be compared with Fig. 3 taken from Grossmann's calculations for which the criterion of equal half temperature times was used.

The use of curve a in Fig. 2 may be illustrated by an example. Let us assume that the severity of quench, H, has been measured or estimated and is equal to 2. Assuming that it has also been determined experimentally by an etch test or hardness survey that

-5-

a 2.5" round bar of a given steel will harden throughout when quenched in this medium, we wish to know what size round bar would harden in a medium of $H = 1$, and $H = \infty$ (ideal medium). It is first necessary to multiply H by D_{SH}, which $= 5$. From Fig. 2a is read the value of the ratio D_{SH}/D_S (.81). We obtain $D_{S\infty}$ by dividing D_{SH} (2.5") by this ratio. The ideal diameter is therefore equal to 3.1". A 3.1" diameter bar of the given steel would harden throughout in a medium of infinite severity of quench. To obtain the diameter which would harden through in a medium of severity $H = 1$, the value of $D_{SH}/D_{S\infty}$ is first read from Fig. 2a. For a value of $HD_{SH} = 2.5$ this ratio is equal to .71. The ratio of $D_{SH=1}/D_{SH=2}$ is found by dividing .71 by .81. This fraction when multiplied by 2.5" ($D_{SH=2}$) is equal to the diameter of the bar which would just harden through in a medium of $H = 1$.

The relation between the thickness of a plate quenched in a medium, H, and the ideal plate thickness is presented as curve b in Fig. 2. This curve is also based on the equivalence of the "c" rather than on the equivalence of half temperature times. The ideal round size equivalent to the ideal plate thickness may be determined from the following relation. (Appendix)

$$D_{S\infty} = 1.53 L_\infty$$

where L_∞ is the ideal plate thickness.

The calculation of the ideal round diameter equivalent to a hollow cylinder is more complicated, for the outside and inside diameters can be varied independently. The equivalent round diameter is determined for a given wall ratio and for a severity of quench measured by the product of H times the difference between the inside and outside diameters.

The results of calculations (Appendix) based on this new criterion are presented as Fig. 4 in which the ratio of equivalent round diameter (D_{SH}) to outside diameter (D_0) is plotted as a function of D_i/D_0 for different values of H times (D_0-D_i) where D_i is the inside diameter. D_{SH} is the diameter of the round which has the same cooling rate at its center as the minimum cooling rate in the wall of the hollow cylinder when quenched in a medium of severity H. The use of curve a of Fig. 2 and Fig. 4 can be best understood through an example; let us assume that the equivalent ideal diameter of a round is to be determined for the following situation:

Severity of quench (H) $= 1$
Outside diameter (D_0) $= 4"$
Inside diameter (D_i) $= 2"$

D_i/D_0 $= 0.5$
D_0-D_i $= 2"$
$H(D_0-D_i)$ $= 2$

The value of D_{SH}/D_0 corresponding to a wall ratio of 0.5 and an $H(D_0-D_i)$ value of 2 is read from Fig. 4. D_{SH}/D_0 in this case is 0.43 and when multiplied by

-7-

D_o(4"), an equivalent round diameter of 1.72" is obtained. By employing the graph of Fig. 2a and the product D_{SH}(1.72) and H(1), $D_{SH}/D_{S\infty}$(0.63) is found. D_{SH} divided by this ratio fixes the ideal round diameter equivalent to the hollow cylinder (2.7").

B. Experimental

In order to determine how closely the theoretical results outlined in the previous section approximate the actual relationships between the sizes of hollow cylinders and rounds having the same minimum hardness, experiments described in this section have been performed.

The ideal critical diameter of a steel of the following composition was calculated by the method of Grossmann.

C	Mn	Si	S	P	Cr	Cu	Mo	Al
.40	.79	.43	.070	.018	.14	.105	.16	.004

The critical diameter in mildly agitated water was measured by quenching from 1575° F (neutral atmosphere) rounds of various diameters from $1\frac{1}{4}$" to $2\frac{1}{4}$" all machined from a $2\frac{5}{4}$" round bar. In all cases the length of the solid cylinder was 4 times the diameter. The water in the quenching tank was agitated by a stream of water entering from the bottom through a $\frac{1}{4}$" diameter aperture at about 25 gal. per minute. The diameters of the unhardened cores were determined after sectioning

-8-

by etching with a hot sulfuric-hydrochloric acid mixture. The critical diameter of the steel and the H value of the medium were then determined by the method of Grossmann and were found to be 1.5" and 1.4 respectively. The ideal diameter calculated from the composition was 2.2" while the measured diameter was equal to 2.1". The ratio D_i/D_o of the hollow cylinder with outside diameter of $2\frac{1}{4}$" and for both inside and outside quenched which would be equivalent to this solid round (1.5") was read from Fig. 4 and D_i/D_o is equal to .3. The wall ratio (D_i/D_o) for the equivalent hollow cylinder for the case in which only the outside is quenched was found from Fig. 4 to be .6. Five hollow cylinders with wall ratios ($D_o = 2.5$") less than each of these two critical values were machined from the $2\frac{1}{2}$" round stock. One set of five hollow cylinders was quenched from 1575° F into the water and supported directly over the aperture. Thin plates were welded over the ends of the second set of hollow cylinders and these were quenched as nearly like the solid cylinders as possible.

The Rockwell "C" hardness distribution for each of the hollow and solid cylinders was measured and the minimum hardnesses are plotted in Fig. 6. A typical hardness survey for both inside and outside quenching is presented in Fig. 7. The size of the round which

-9-

would have the same minimum hardness as each of the
hollow cylinders was determined from Fig. 6. The
experimental determinations and the corresponding
theoretical values read from Fig. 4 are tabulated
in Table I. The difference between theory and ex-
periment for the hollow cylinders with only the out-
side quenched is less than 6 percent. For the case
of both inside and outside quenching the difference
is about 15 percent, the theory predicting a larger
equivalent round size. For these small size cylinders,
the inside quenching is more efficacious than the
outside for the particular conditions of our experi-
ment. In these experiments, it appears that the
severity of quench is larger on inside than the out-
side and the equivalent solid cylinder is necessarily
smaller than that predicted assuming that the inside
and outside of the hollow cylinder were quenched
identically.

TABLE I

Comparison of Theoretical and Experimental Equiva-
lent Solid Cylinders (For Same Minimum Hardness)

| Hollow Cylinder Only Outside Quenched | | Diameter of Equivalent Solid Cylinder (Inches) | |
D_1 (Inches)	D_0 (Inches)	Exp.	Theo.
1.00	2.48	1.94	1.93
1.13	2.48	1.70	1.81
1.25	2.46	1.57	1.66
1.38	2.48	1.44	1.53
1.50	2.48	1.38	1.38

Both Inside & Outside Quenched			
.38	2.48	1.55	1.75
.50	2.48	1.44	1.69
.63	2.48	1.31	1.54
.75	2.48	1.20	1.45
.88	2.48	1.10	1.35

APPENDIX A

CALCULATIONS

1. General Theory

The purpose of this section is to find the relation
between the equivalent ideal critical diameter $(D_{S\infty})$ of
a solid cylinder and of a hollow cylinder of inside di-
ameter D_i and outside diameter D_o, when quenched in some
medium. This relation will contain as parameters the
severity of the quench in the actual medium, H, and the
wall ratio (a) of the inside diameter over the outside
diameter. Thus

$$D_S = F(a,H,D_o).$$

The mathematical problem to be solved is the de-
termination of the constant c (Eq.1) for hollow and for
solid cylinders, for various severities of quench.

The differential equation for heat flow is

$$\partial T/\partial t = k\nabla^2 T. \qquad (2)$$

In this equation, k is the thermal diffusion coefficient,
∇^2 is the LaPlacian operator. Since we are considering
the special case in which Eq. (1) is satisfied, we shall
set

$$T = T_o \left\{ \exp(-ct) \right\} U(r). \qquad (3)$$

Here, as in Eq. (1), T is the temperature of element
referred to that of the quenching medium, U is an unknown

function of coordinates, and c is a constant to be determined.

Substitution of Eq. (3) into Eq. (2) gives Eq. (5) below as the differential equaltion for $U(r)$; with
$$m^2 = c/k. \tag{4}$$

Only those solutions of this differential equation are desired which satisfy the appropriate boundary condition. The boundary condition which has been found to be fairly accurate is given by Newton's law of cooling. This law states that the flow of heat across a boundary is proportional to the temperature drop across that boundary. Since the flow of heat is proportional to the temperature gradient, the appropriate boundary condition is that the temperature gradient at the boundary is proportional to the difference in temperature between the solid and the quenching medium. The constant of proportionality is taken as $-h$, the gradient being taken in the direction from the solid to the quenching medium. The constant h is equal to twice the H used by Grossmann. This boundary condition may be written as in Eq. (6). Eqs. (5) and (6) below form the complete mathematical formulation of the problem.

Differential equation for U:
$$\nabla^2 U = -m^2 U. \tag{5}$$

Boundary condition for U:

$\text{gradient}_n U = -hU$ at surface. (6)

2. Plates and Solid Cylinders

The solution of these equations have previously been given for the cases of a plate and of a solid cylinder. In the case of the plate, of thickness 2L,

$$U = \cos mx,$$

where m is the lowest root of the equation

$$\tan mL = h/m. \tag{7}$$

In the case of a solid cylinder of radius r_s

$$U = J_0(mr),$$

where m is the lowest root of the equation

$$J_1(mr_s)/J_0(mr_s) = h/m. \tag{8}$$

Eqs. (7) and (8) have no solution for arbitrary values of m, but only for a discrete set of values. In the present case one is interested only in the lowest value of m. The higher values are important only when the transient effects are being studied.

<u>a</u>. Limit $h = 0$.

In the case of a very poor quenching medium, $h \to 0$ and also $m \to 0$. The left members of Eqs. (7) and (8) may therefore be replaced by the first term in a Taylor expansion. We obtain

$$mL = h/m, \tag{7a}$$

$$\tfrac{1}{2} mr_s = h/m. \tag{8a}$$

The ratio of the diameter of the solid cylinder to

the thickness of equivalent plate is therefore given
by the equation

$$r_s/L = 2. \qquad (9)$$

b. Limit $h = \infty$.

The case of an ideal quench is obtained by setting
$h = \infty$. Eqs. (7) and (8) now become, with $r_{s\infty}$ written
for r_s,

$$mL_\infty = \eta/2, \qquad (7b)$$

$$mr_{s\infty} = 2.404. \qquad (8b)$$

In this case we obtain the ratio

$$r_{s\infty}/L_\infty = 1.53. \qquad (10)$$

The observed value[9] of the ratio r_s/L lies between
the above two extreme values of 1.53 and 2.0, while the
ratio as calculated by Grossmann's method of half
temperature time is 1.57.[3]

By means of Eqs. (8) and (8b), one can obtain the
ratio $r_s/r_{s\infty}$ of the radius of a solid cylinder in an
actual quench to the radius of a solid cylinder in an
ideal quench. From Eq. (8) h/m is obtained as a
function of mr_s. Upon forming the quotient $mr_s/2.404$
we obtain, by Eq. (8b), $r_s/r_{s\infty}$. Upon forming the
product $mr_s \times (h/m)$ we obtain hr_s. The resulting re-
lation between $r_s/r_{s\infty}$ and hr_s is given as Fig. 2. In
this figure the first quantity is written as $D_s/D_{s\infty}$,
the latter quantity as HD_{sm}. In an identical manner, one

can obtain from Eqs. (7) and (7b) the ratio L_H/L_∞
of the thickness of a plate in an actual quench to
the equivalent thickness in an ideal quench. The
result is given as Fig. 3a.

  c. Intermediate quenches.

The ratio r_s/L for intermediate quenches has been
obtained as follows. For a series of values of h/m,
the corresponding values of mL and mr_s were found from
Eqs. (7) and (8), respectively. By forming the ratio
$(mr_s/mL) = r_s/L$, and the product $2(mL) \cdot (h/m) = 2Lh$,
r_s/L was obtained as a function of $2Lh$. The transition
of r_s/L, which will be denoted by $x(h)$, from 2 to 1.53
as h goes from o to ∞, is best presented by a graph
giving $(x-x_\infty)/(x_0-x_\infty)$ as a function of $2Lh$. This
is given as Fig. 5.

3.   Hollow Cylinder, Inside and Outside Quenched

In a hollow cylinder the general solution of Eq. (5)
must be used, namely

$$U(r) = C_0 \, J_0(mr) + C_1 N_0(mr), \qquad (11)$$

where J_0 and N_0 are Bessel functions of the first and
second type, respectively.[11] The two constants must be
so chosen as to satisfy the boundary condition (6) at
both the inner and the outer surfaces.

The boundary condition (6) applied to both the
inner and outer surfaces, gives two simultaneous linear
homogeneous equations in the two unknowns C_0 and C_1.

These equations are soluble only if the determinant
of the coefficients is zero. This condition gives the
following equation, which corresponds to Eq. (8) for a
solid cylinder.

$$\frac{m\ J_0'(mr_0) + h\ J_0(mr_0)}{m\ N_0'(mr_0) + h\ N_0(mr_0)} = \frac{m\ J_0'(amr_0) - h\ J_0(amr_0)}{m\ N_0'(amr_0) - h\ N_0(amr_0)} \ . \quad (12)$$

a. Limit $h = 0$.

One obtains the extreme case of a poor quench by
letting h, and hence also m, approach zero. In this case
each Bessel function may be replaced by the first term
in the Taylor expansion. Using the expansion given in
reference (11), one reduces Eq. (12) to

$$\tfrac{1}{2}\ mr_0(1-a) = h/m. \quad (12a)$$

Upon dividing Eq. (8a) by Eq. (12a) one obtains

$$r_s/r_0 = 1-a \quad (13)$$

as the ratio of the radius of the round to that of an
equivalent hollow cylinder.

b. Limit $h = \infty$.

The extreme case of an ideal quench is obtained by
letting h approach ∞ in Eq. (12). In this case Eq. (12)
reduces to

$$\frac{J_0(mr_0)}{N_0(mr_0)} = \frac{J_0(amr_0)}{N_0(amr_0)} \ . \quad (12b)$$

zero, one obtains the equation

$$\frac{m\, J_0'(mr_0) + h\, J_0(mr_0)}{m\, N_0'(mr_0) + h\, N_0(mr_0)} = \frac{J_0'(a\, mr_0)}{N_0'(a\, mr_0)} \tag{14}$$

a. Limit $h = 0$.

Proceeding as in Section 3a, we retain only the first terms in the Taylor expansion of the functions in Eq. (14). In this case Eq. (14) reduces to

$$\tfrac{1}{2}\, mr_0\, (1-a^2) = h/m. \tag{14a}$$

Upon comparing this equation with Eq. (8a), we obtain

$$r_s/r_0 = (1-a^2).$$

This relation is given in Fig. 4.

b. Limit $h = \infty$.

In the case of an ideal quench, $h = \infty$, Eq. (14) reduces to

$$\frac{J_0(mr_0)}{N_0(mr_0)} = \frac{J_0(amr_0)}{N_0(amr_0)}. \tag{14b}$$

This equation has been solved graphically in the same manner as was Eq. (12b). The result is given in Fig. 4.

c. Intermediate Quenching Rates

When $a = 1$, the distribution of temperature in the hollow cylinder becomes identical to the temperature

Since m may be replaced by $2.404/r_{s\infty}$ from Eq. (8b), the ratio $r_{s\infty}/r_o$ may be determined as a function of $\underline{a}$. This function has been determined by a graphical method, and is given in Fig. $\underline{4}$.

c. Intermediate Quenching

The variation of the ratio $x = r_s/r_o$ with severity of quench may best be studied through the ratio $(x-x_\infty)/(x_o-x_\infty)$. At least in the case of a thin hollow cylinder, a = 1, the dependence of this ratio upon $h\ (r_o-r_i)$ will be given by the graph in Fig. 5 which was derived originally from a comparison of rounds with plates in Section 2 c. A numerical calculation for r_s/r_o was carried out for the cases a = 0.1 and a = 0.5. The results showed that Fig. 5 was applicable to values of $\underline{a}$ at least as low as 0.1. The intermediate lines in Fig. $\underline{4}$ have been based upon Fig. $\underline{5}$.
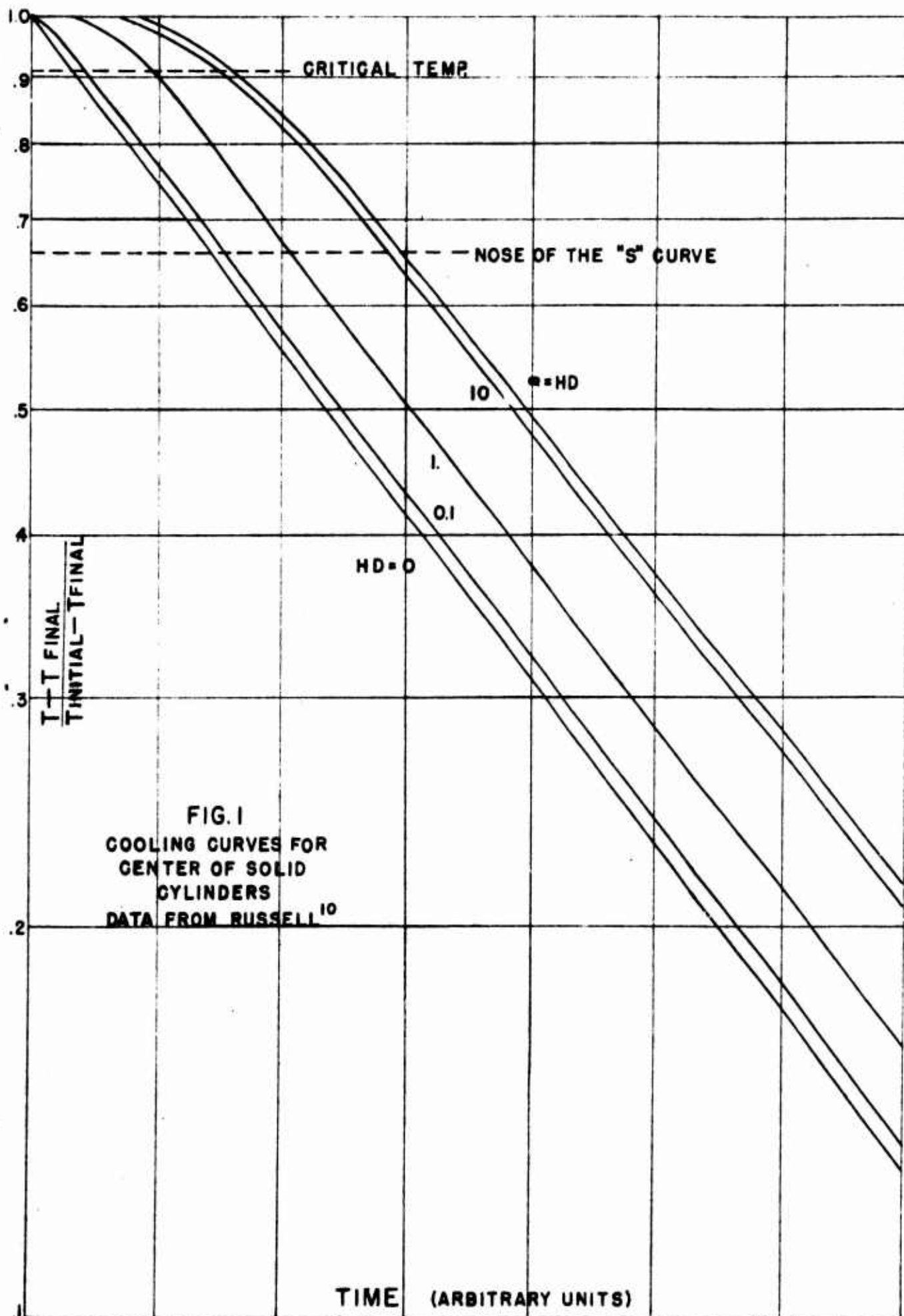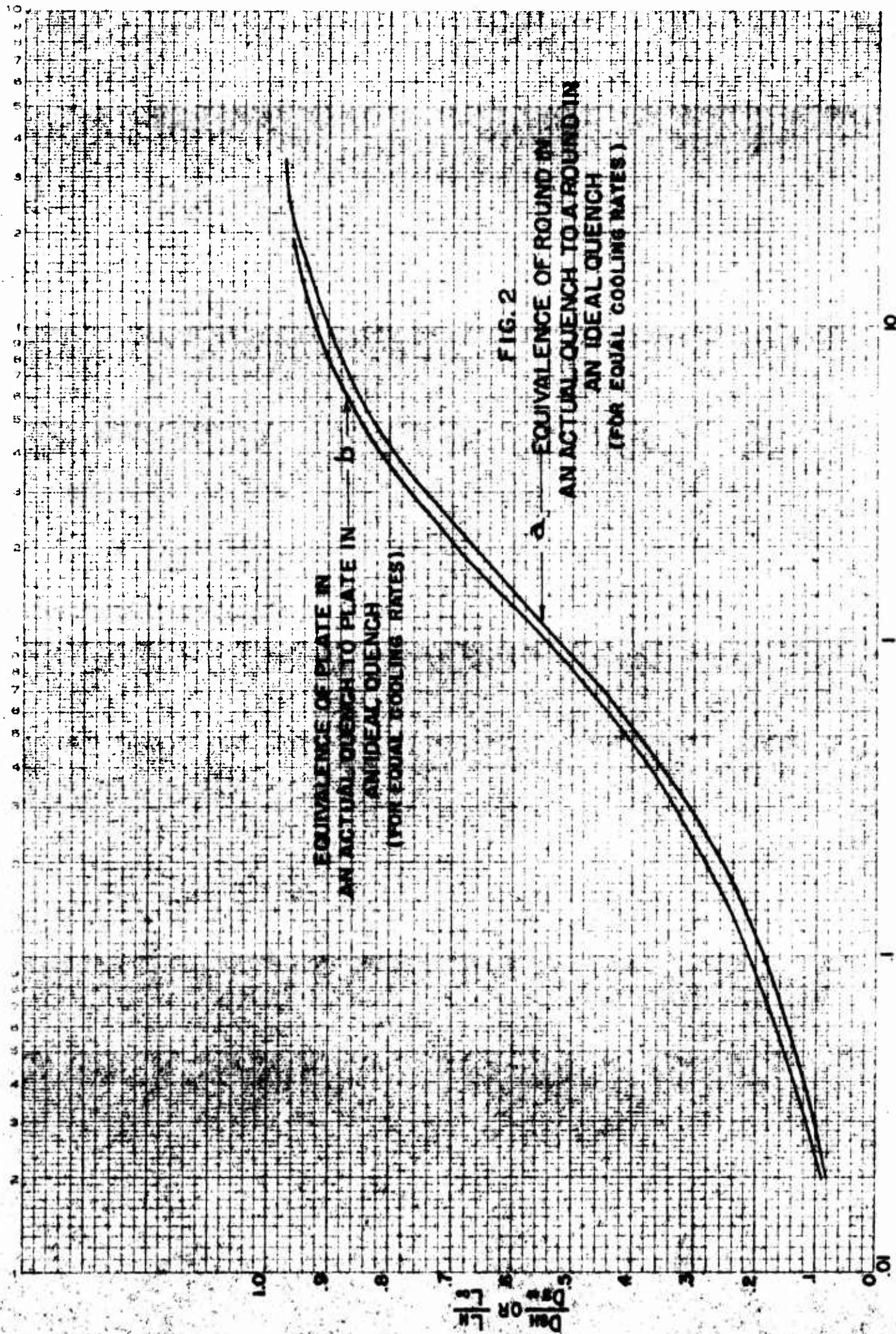
4. Hollow Cylinder, Only Outside Quenched

In the case of a hollow cylinder quenched only on the outside, we must again use the general solution given in Eq. (11). The constants C_o and C_1 are to be so chosen that the boundary condition (6) is satisfied with $\underline{h}$ set equal to zero at the inner boundary. The two conditions corresponding to the two boundaries lead to two linear homogeneous equations in C_o and C_1. These equations are soluble only when the determinant of the coefficients is zero. Upon equating this determinant to

distribution throughout one-half of a plate quenched on both sides, whose thickness is double the real thickness of the hollow cylinder. The transition from $h \rightarrow 0$ to $h \rightarrow \infty$ is therefore obtained from Fig. 5 provided the abscissa is now $2H \cdot (D_o - D_i)$ in place of $H(D_o - D_i)$.

BIBLIOGRAPHY

1. M. Asimow and M. A. Grossmann: Trans. A. S. M.,
    Vol. 28, 1940, p. 949.

2. M. A. Grossmann: T. P. 1437, A. I. M. E.,
    February, 1942.

3. M. A. Grossmann, M. Asimow, and S. E. Urban:
    Hardenability of Alloy Steels, A. S. M. 1939.

4. W. E. Jominy: Metal Progress, December, 1941.

5. M. Asimow and M. A. Grossmann: S. A. E. Journal,
    Vol. 49, No. 1, p. 266.

6. M. A. Grossmann and M. Asimow: The Iron Age,
    April 25-May 2, 1940.

7. A. E. Focke: The Iron Age, Vol. 150, No. 9, p. 43.

8. M. Asimow, A. Craig, and M. A. Grossmann: S. A. E.
    Journal (1941) 283.

9. C. Post and O. Green: Trans. A. S. M., Vol. 28,
    1940, p. 962.

10. T. F. Russell: First Report of the Alloy Steels
    Research Committee (Iron and Steel Institute)
    1936, p. 149.

11. E. Jahnke and F. Emde: Funcktionentafeln.
    (Berlin 1928)

FIG. I
COOLING CURVES FOR
CENTER OF SOLID
CYLINDERS
DATA FROM RUSSELL[10]

EQUIVALENCE OF PLATE IN
AN ACTUAL QUENCH TO PLATE IN
AN IDEAL QUENCH
(FOR EQUAL COOLING RATES)

EQUIVALENCE OF ROUND IN
AN ACTUAL QUENCH TO A ROUND IN
AN IDEAL QUENCH
(FOR EQUAL COOLING RATES)
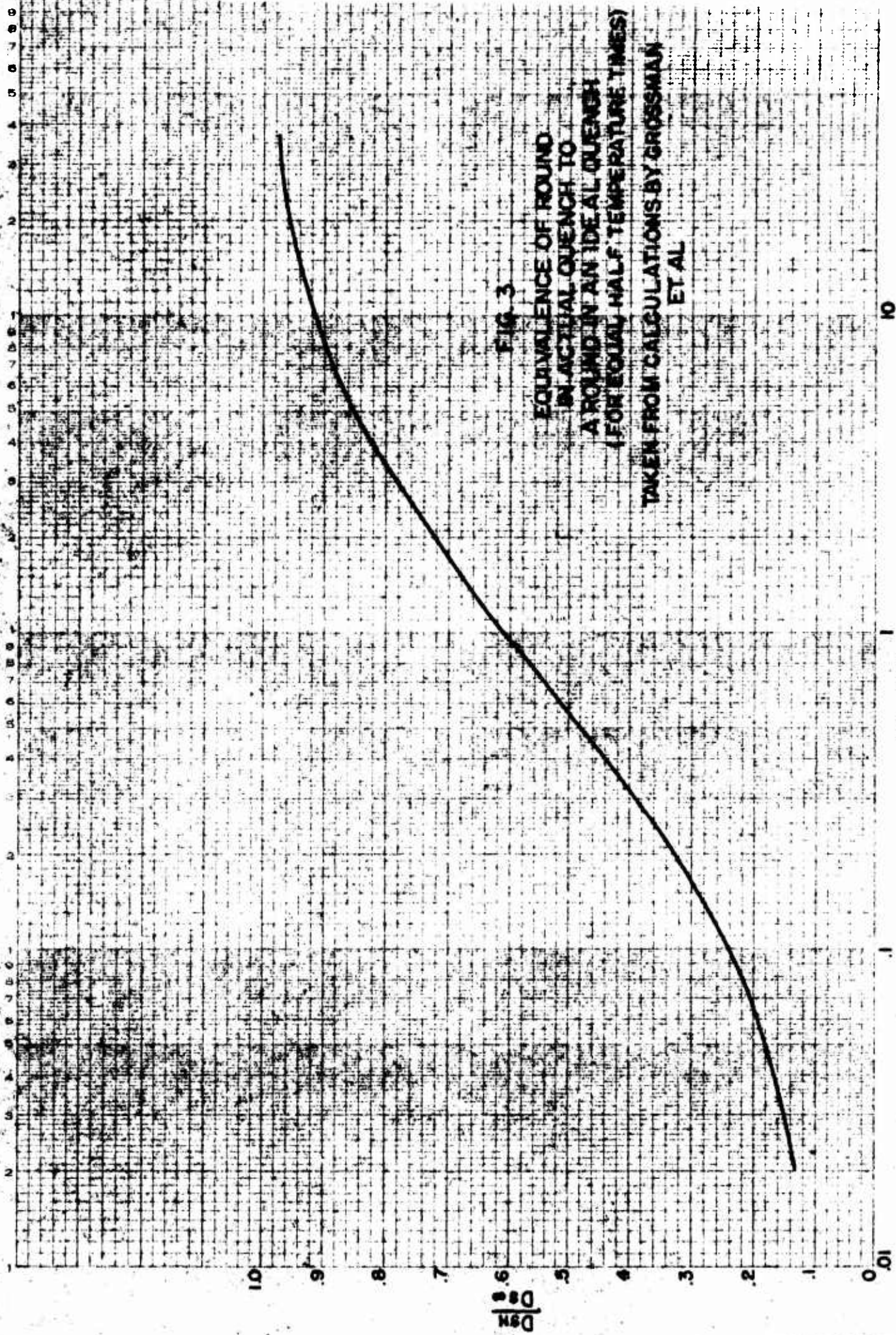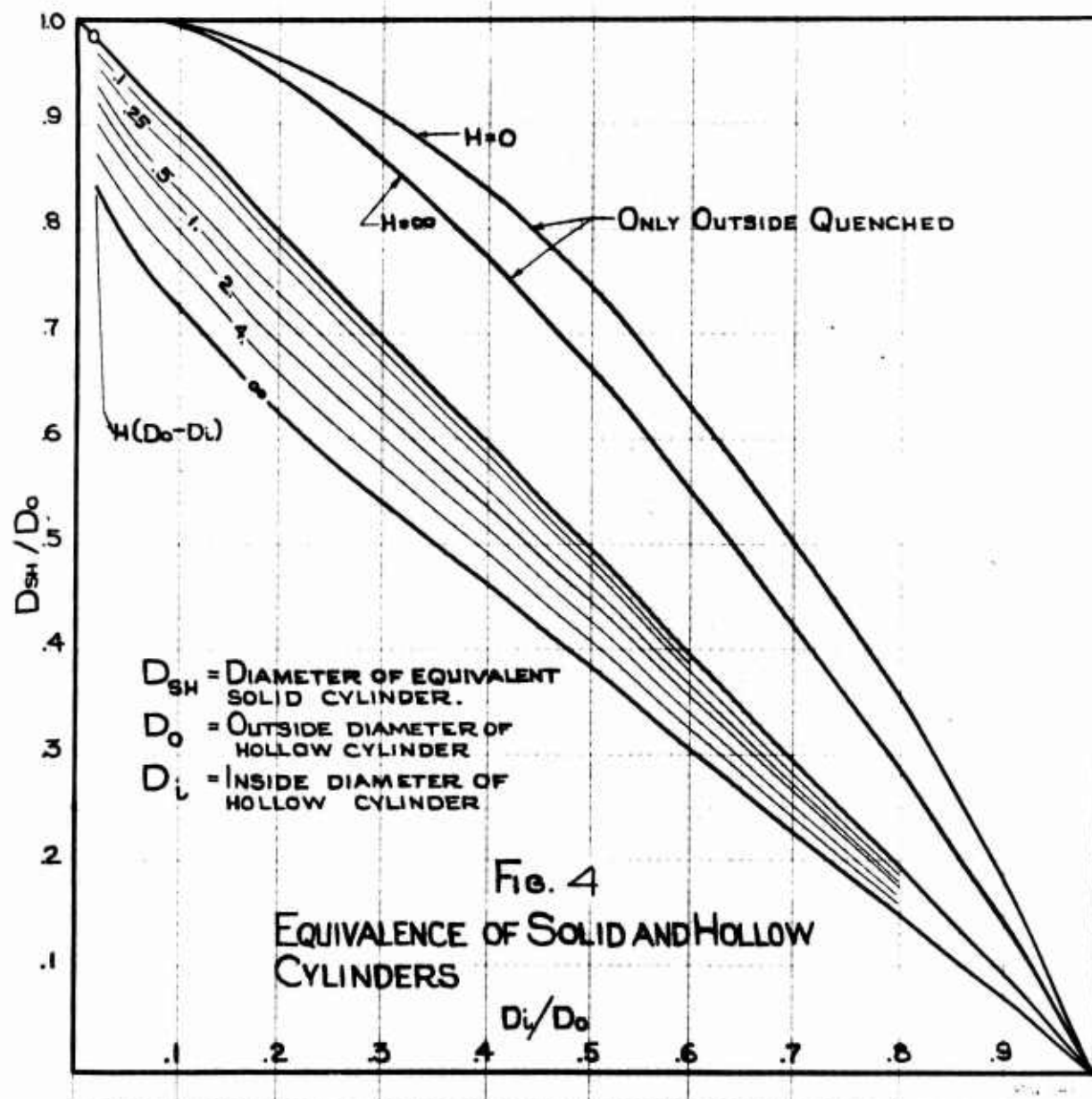
FIG. 2

H Dsh OR H 2 Lm

FIG. 3

EQUIVALENCE OF ROUND
IN ACTUAL QUENCH TO
A ROUND IN AN IDEAL QUENCH
(FOR EQUAL HALF TEMPERATURE TIMES)

TAKEN FROM CALCULATIONS BY GROSSMAN,
ET AL

Fig. 4

EQUIVALENCE OF SOLID AND HOLLOW CYLINDERS

D_{SH} = DIAMETER OF EQUIVALENT SOLID CYLINDER.

D_o = OUTSIDE DIAMETER OF HOLLOW CYLINDER

D_i = INSIDE DIAMETER OF HOLLOW CYLINDER

FIG. 5

TRANSITION FROM POOR
TO IDEAL QUENCH

X=Ds/2L — FOR PLATE
X=Ds/(Do−Di) — FOR HOLLOW
CYLINDER

Axis labels:
- Vertical: $(X-X_\infty)/(X_0-X_\infty)$
- Horizontal: 2HL (WHEN X=Ds/2L−PLATE)   H(Do−Di) (WHEN X=Ds(Do−Di)−HOLLOW CYLINDER)
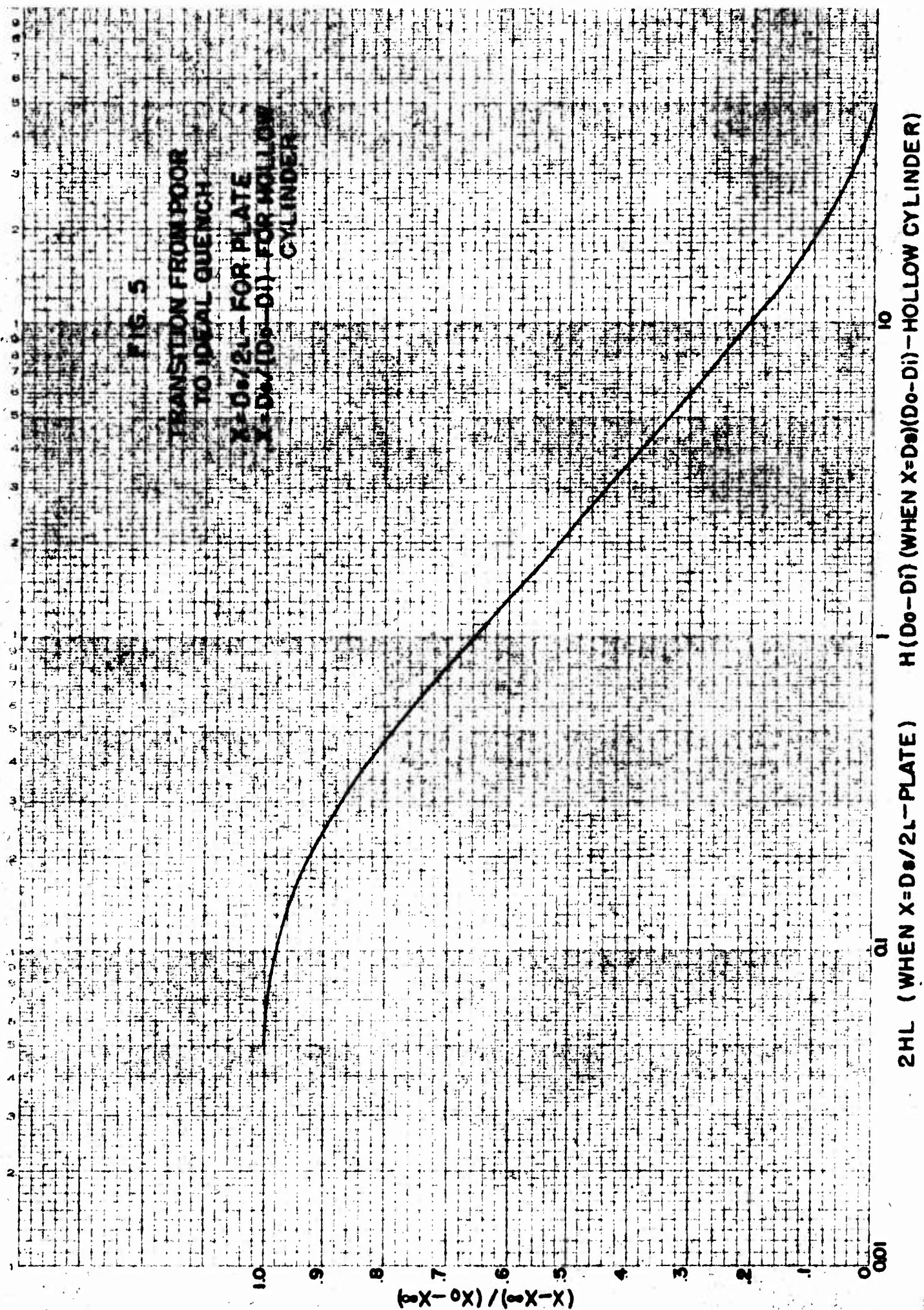
FIG. 6
MINIMUM HARDNESS OF
SOLID & HOLLOW CYLINDERS
AS A FUNCTION OF DIMENSION
(EXPERIMENTAL)

HOLLOW CYLINDERS
QUENCHED ONLY OUTSIDE

HOLLOW CYLINDERS
QUENCHED OUT-
SIDE & INSIDE

SOLID CYLINDERS

ROCKWELL "C" HARDNESS

DIAMETER OF HOLE OF HOLLOW CYLINDER (D_0=2.48") OR DIAMETER
OF SOLID CYLINDER

FIG. 7

TYPICAL HARDNESS SURVEY
FOR HOLLOW CYLINDER
QUENCHED INSIDE & OUTSIDE